CBSE Board
Class XII Mathematics
Board Paper 2012
Delhi Set – 2

Time: 3 hrs **Total Marks: 100**

General Instructions:

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three Section A, B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.
3. All questions in section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six mark each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

1. Evaluate $\int (1-x)\sqrt{x}\,dx$.

2. Evaluate: $\int_2^3 \frac{1}{x}\,dx$

3. If $\begin{pmatrix}2 & 3\\ 5 & 7\end{pmatrix}\begin{pmatrix}1 & -3\\ -2 & 4\end{pmatrix}=\begin{pmatrix}-4 & 6\\ -9 & x\end{pmatrix}$, write the value of x.

4. Find 'λ' when the projection of $\vec{a}=\lambda\hat{i}+\hat{j}+4\hat{k}$ on $\vec{b}=2\hat{i}+6\hat{j}+3\hat{k}$ is 4 units.

5. If a line has direction ratios 2,-1,-2 then what are its direction cosines?

6. Let * be a 'binary' operation on N given by a * b = LCM (a, b) for all a, b $\in$ N. Find 5 * 7.

7. Write the principal value of $\cos^{-1}\left(\frac{1}{2}\right)-2\sin^{-1}\left(-\frac{1}{2}\right)$.

8. Simplify: $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$

9. Find the sum of the following vectors:
$\vec{a}=\hat{i}-2\hat{j},\ \vec{b}=2\hat{i}-3\hat{j},\ \vec{c}=2\hat{i}+3\hat{k}.$

10. If $\Delta=\begin{vmatrix}5 & 3 & 8\\ 2 & 0 & 1\\ 1 & 2 & 3\end{vmatrix}$, white the cofactor of the element a_{32}.

SECTION – B

11. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $|\vec{a}|=5, |\vec{b}|=12$ and $|\vec{c}|=,\ 13$ and $\vec{a}+\vec{b}+\vec{c}=0$ Find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$.

12. Solve the following differential equation:

$2x^2\frac{dy}{dx}-2xy+y^2=0.$

13. How many times must a man toss a fair coin, so that the probability of having at least one head is more than 80%?

14. If $(\cos x)^y = (\cos y)^x$, find $\frac{dy}{dx}$.

OR

If $\sin y = x\sin(a+y)$, prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$.

15. Let A = R – {3} and B = R – {1}. Consider the function f : A → B defined by $f(x)=\left(\frac{x-2}{x-3}\right)$. Show that f is one-one and onto and hence find f^{-1}.

16. Prove that $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)=\frac{\pi}{4}-\frac{\pi}{2}, x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

OR

Prove that $\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\cos^{-1}\left(\frac{36}{85}\right)$.

17. Find the point on the curve $y = x^3 - 11x + 5$ at which the equation of tangent is $y = x - 11$.

OR

Using differentials, find the approximate value of $\sqrt{49.5}$.

18. Evaluate: $\int \sin x \sin 2x \sin 3x \, dx$

OR

Evaluate: $\int \frac{2}{(1-x)\left(1+x^2\right)} dx$

19. Using properties of determinants prove the following:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

20. If $y = 3\cos(\log x) + 4\sin(\log x)$, show that

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

21. Find the equation of the line passing through the point (-1,3,-2) and perpendicular to the lines $\frac{x}{1} = \frac{y}{2} \; \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$.

22. Find the particular solution of the following differential equation:

$(x+1)\frac{dy}{dx} = 2e^{-y} - 1; y = 0$ when $x = 0$.

SECTION – C

23. Using matrices solve the following system of linear equations:

$$x - y + 2z = 7$$
$$3x + 4y - 5z = -5$$
$$2x - y + 3z = 12$$

OR

Using elementary operations, find the inverse of the following matrix:

$$\begin{pmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$$

24. A manufacturer produces nuts and bolts. It takes 1 hours of work on machine A and 3 hours on machine B to product a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of ₹17.50 per package on nuts and ₹7 per package of bolts. How many packages of each should be produced each day so as to maximize his profits if he operates his machines for at the most 12 hours a day? From the above as a linear programming problem and solve it graphically.

25. Find the equation of the plane determined by the point A(3, - 1, 2), B(5, 2, 4) and C(-1, - 1, 6) and hence find the distance between the plane and the point P(6, 5, 9).

26. Prove that $\int_0^{\frac{\pi}{4}} \sqrt{\tan x} + \sqrt{\cot x}\, dx = \sqrt{2} \cdot \frac{\pi}{2}$.

OR

Evaluate $\int_1^3 \left(2x^2 + 5x\right) dx$ as a limit of sum.

27. Show that the height of a closed right circular cylinder of given surface and maximum volume, is equal to the diameter of its base.

28. A girl throws a cie. If she get a 5 OR 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 OR 4, she tosses a coin two times and notes the number of heads obtained. If she obtained exactly two heads, what is the probability that she threw 1, 2, 3 OR 4 with the die?

29. Using the method of method of integration, find the area of the region bounded by the following lines:

$3x - y - 3 = 0$,

$2x + y - 12 = 0$,

$x - 2y - 1 = 0$