MADHYAMIK 2012 (Mathematics)

[ENGLISH VERSION]

(Bifurcated Syllabus)

The answers of the questions of PART I are to be given at the beginning of the answer script mentioning the question numbers in the serial order. Necessary calculations and drawing, if any, must be given in the right hand side by drawing margin at the first few pages in the answer script. Tables or calculator cannot be used. If necessary, take $\pi = \frac{22}{7}$. Graph paper will be supplied as required.]

(Alternative questions of 7 and 13 are given at page no. 17 for sightless candidates.)

PART I

The answers of all questions of this part are to be written at the beginning of the answer script

1. Answer all questions :

(i) Write down the ratio of 10 days and 2 months. 1

(ii) If $0 < x < 1$ then the L.C.M. of x^6, x^2, x^3 will be :

(a) x^6 (b) x^3 (c) x^{11} (d) x^2 1

(iii) For what value of k the simultaneous equations

$3x + 4y = 18$; $kx - 4y = 180$ have no solution ? 1

(iv) Complete the following sentence with correct answer: 1

Distance between the points (0, 4) and (0, -3) is

(a) 1 unit (b) 7 unit (c) 5 unit (d) -5 unit

(v) PQRS is a cyclic quadrilateral and $\angle PQR = 80^o$ what is the value of $\angle RSP$? 1

(vi) What is the simplest value of $\tan 70^o \times \tan 20^o$? 1

2. Answer all questions :

(i) The average of first 21 natural numbers is 11; then find the average of first 20 natural numbers. 2

(ii) If $x : y = 3 : 4$ find $(3y - x) : (2x + y)$. 2

(iii) Find the greatest and least value of x from the inequations $19 \leq 2x - 1 \leq 29$. 2

(iv) ABCD is a parallelogram and $\angle B - \angle C = 60^o$; find $\angle A and \angle D$. 2

(v) Δ ABC is equilateral and AD is median. G is the centroid and GD = 10 cm, find the length of GB. 2

(vi) If v, e and s are the numbers of vertices, edges and plane surfaces respectively of a pyramid with rectangular base then find the value of v + s - e. 2

(vii) If $\tan\theta + \sec\theta = \frac{1}{2}$ then find $\sec\theta$ 2

3. **Answer any TWO questions (Algebraic methods may be applied):** 2x5=10

(a) The ratio of syrup and water in a container is 3 : 5 and in other container the ratio is 6 : 1. In what ratio the mixtures of two containers be mixed so that the ratio of syrup and water will be 4 : 3 in the new mixture ?

(b) Charging 30% above the production cost Madan Tanti puts a label of Rs. 8,580 on a Baluchari Sari. But at the time of selling he allows 10% discount. Find his gain in percent.

(c) A person kept Rs. 18,750 for his two sons of age 12 years and 14 years. It is decided that the respective money will be deposited at 5% simple interest and when the boys will reach 18 years they will get equal amount of money. Find the amount of money kept for two sons individually.

(d) Present value of a printing machine is Rs. 1,80,000. Value of the machine depreciates at the rate of 10% every year. What will be the value of the machine after 3 years ?

4. **Answer any ONE question:** 4

(a) Find the H.C.F. of $x^2 + 3x + 2$, $3x^2 + x - 2$ and $2x^3 - x^2 - 3x$.

(b) Find the L.C.M.of $15a^3 (a + x)^3$, $20ax^3 (a - x)^3$ and $36a^2 x^2(a^2 - x^2)^2$.

5. **Solve (any ONE):** 3

(a) $3x - 4y = 1$; $4x = 3y + 6$ [by the method of elimination or cross multiplication]

(b) $(x -7)(x - 9) = 195$.

6. Answer any ONE question : 4

(a) The sum of the digits of a two digit number is 7; if 27 be added to the number, the digits of the number interchange their places. Find the number.

(b) Area and perimeter of a rectangular park are 600 sq. m and 100 m respectively. Find the length and breadth of the park.

7. Draw the graphs of the inequations and indicate the solution region (any ONE): 4

(a) $x \geq -3$; $y \geq 0$ and $x + y \leq 10$

(b) $y \leq 0$; $3x + 2y + 6 \geq 0$ and $3x - 2y \leq 6$.

8. **Answer any ONE question :** 3

(a) Given $\frac{x+y}{x-y} = \frac{a}{b}$, prove that $\frac{y^2+xy}{x^2-xy} = \frac{a^2-ab}{b^2+ab}$

(b) If $a:b = b:c$, then show that $abc(a+b+c)^3 = (ab+bc+ca)^3$.

9. Answer any ONE question : 3

(a) Volume of a sphere varies directly as cube of its radius. Diameter of a solid sphere of lead is 14 cm. Applying theory of variation prove that the number of spheres of diameter 7 cm made by melting this sphere is 8. (Assume that no volume is lost for melting).

(b) Given $a \propto b$ and $b \propto c$ show that $a^3+b^3+c^3 \propto 5abc$.

10. Answer any ONE question : 3

(a) If $a = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $b = \frac{\sqrt{5}-1}{\sqrt{5}+1}$, then find the value of $\frac{a^2+ab+b^2}{a^2-ab+b^2}$

(b) Simplify:

11. Answer any TWO questions : 2 x 5=10

(a) Prove that a straight line, drawn from the centre of a circle to bisect a chord which is not a diameter is at right angles to the chord.

(b) Prove that if two circles touch each other externally, then the point of contact will lie on the line joining the two centres.

(c) "If a perpendicular is drawn from the vertex containing the right angle of a right angled triangle on the hypotenuse, the triangles on each side of the perpendicular are similar to one another."— Prove.

12. Answer any ONE question : 3

(a) In the cyclic quadrilateral *ABCD*, *AB* = *DC*, prove that *AC* = *BD*.

(b) *ABCD* is a trapezium such that *AB* || *CD*. Its diagonals *AC* and *BD* intersect each other at O. Prove that *AO.OD* = *BO.OC*.

13. Answer any ONE question : 5

(a) Draw $\triangle ABC$ such that *AB* = *8* cm, *BC* = *6* cm and $\angle ABC = 60^o$; then draw the circumcircle of the triangle.

(Only traces of construction are required.)

(b) Find geometrically the value of $2\sqrt{7}$.

(Only traces of construction are required.)

14. Answer any ONE question : 3

(a) The base of a pyramid is a square of side 12 cm and its volume is 576 cubic cm. Find its slant height.

(b) Area of the curved surface of a sphere is 5544 sq. cm. Calculate its volume.

15. Answer any ONE question : 4

(a) A hemisphere and a cone are on equal bases and their heights are also equal. Find the ratio of their volumes and ratio of their curved surfaces.

(b) Diameter of the cross section of a wire is decreased by 50%. How much percent will the length be increased so that the volume remains same ?

16. Answer any TWO questions : 2 x 3 = 6

(a) In a right angled triangle the difference between two acute angles is 30°. Express these two angles in degrees and radians.

(b) $\tan A = \frac{x}{y}$, find the value of $\frac{\cos A - \sin A}{\cos A + \sin A}$

(c) Given $x\sin 60^o \cos^2 30^o = \frac{\tan^2 45^o \sec 60^o}{\mathrm{cosec}\, 60^o}$, find the value of x.

(d) In a triangle ABC, prove that $\sin\frac{A+B}{2} + \cos\frac{B+C}{2} = \cos\frac{C}{2} + \sin\frac{A}{2}$.

17. Answer any ONE question : 5

(a) Apu standing on the deck of a ship, which is 10 m above the water level, observes the angle of elevation of the top of a lighthouse as 60° and the angle of depression of the base of the lighthouse as 30°. Calculate the distance of the lighthouse from the ship and the height of the lighthouse.

(b) Durga standing on a railway overbridge $5\sqrt{3}$ m high observes the engine of a running passenger train at an angle of depression of 30°. But just after 2 seconds, she observes the engine at an angle of depression of 60° on the other side of the overbridge. Durga stands vertically above the railway track which is a straight line. Determine the velocity of the train.

[Alternative Questions for Sightless Candidates]

7. Find co-ordinates of two points each on both the graphs of the following equations :

3x + 2y + 6 = 0 and x + y = 10. 4

13. Answer any ONE question : 5

(a) Describe the process of drawing a triangle whose two sides and the included angle are given. Describe also the process of drawing circumcircle of the triangle.

(b) Describe the process of finding the square root of 28 geometrically.

[Additional questions for External Candidates]

18. Answer all questions :

(i) Rate of profit on cost price is 25%. Selling price of a dhoti is Rs. 75. Find the cost price. 2

(ii) x is the mean proportional of -a and -b. Find the values of x. (a and b are positive integers) 2

(iii) x and y are positive integers, and x > y . Among the following statements indicate those which are correct :

(a) $-x < y$ (b) $\frac{1}{x} > \frac{1}{y}$,(c) $-\frac{1}{x} > -\frac{1}{y}$ (d) $x^2[?][?] < [?][?]y^2$ 2

(iv) *AP* is a tangent at *P* to a circle having centre at *O*. *OP* = 3 cm, *AP* = 5 cm;

find the length of *AO*. 2

(v) What is the whole surface area of a solid hemisphere of radius x cm ? 1

(vi) Find the value of sin 60° tan 45° cos 90°. 1