MATHEMATICS

SECTION A

Question 1 [10 × 3]

(i) If $(A - 2I)(A - 3I) = 0$, where $A = \begin{pmatrix} 4 & 2 \\ -1 & x \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, find the value of x.

(ii) Find the value(s) of k so that the line $2x + y + k = 0$ may touch the hyperbola $3x^2 - y^2 = 3$.

(iii) Prove that: $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\sin^{-1}\frac{4}{5}$

(iv) Using L'Hospital's Rule, evaluate:

$$\lim_{x \to 0}\left(\frac{e^x - e^{-x} - 2x}{x - \sin x}\right)$$

(v) Evaluate: $\int \frac{1}{x + \sqrt{x}}dx$

(vi) Evaluate: $\int_0^1 \log\left(\frac{1}{x} - 1\right)dx$

(vii) Two regression lines are represented by $4x + 10y = 9$ and $6x + 3y = 4$.
Find the line of regression of y on x.

(viii) If 1, w and w^2 are the cube roots of unity, evaluate $(1 - w^4 + w^8)(1 - w^8 + w^{16})$

(ix) Solve the differential equation:

$$log\left(\frac{dy}{dx}\right) = 2x - 3y$$

(x) If two balls are drawn from a bag containing three red balls and four blue balls, find the probability that:

(a) They are of the same colour.

(b) They are of different colours.

Question 2

(a) Using properties of determinants, prove that: [5]

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ y+z & z+x & x+y \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$$

(b) Find A^{-1}, where $A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -2 \end{bmatrix}$ [5]

Hence, solve the following system of linear equations:

$4x + 2y + 3z = 2$

$x + y + z = 1$

$3x + y - 2z = 5$

Question 3

(a) Solve for x: $\sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x$ [5]

(b) Construct a circuit diagram for the following Boolean Function: [5]

(BC+A)(A'B'+C') + A'B'C'

Using laws of Boolean Algebra, simplify the function and draw the simplified circuit.

Question 4

(a) Verify Lagrange's Mean Value Theorem for the function $f(x) = \sqrt{x^2 - x}$ in the interval [1, 4]. [5]

(b) From the following information, find the equation of the Hyperbola and the equation of its Transverse Axis: [5]

Focus: (−2, 1), Directrix: $2x - 3y + 1 = 0$, $e = 2/\sqrt{3}$

Question 5

(a) If $y = (\cot^{-1} x)^2$, *show that* $(1+x^2)^2 \frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx} = 2$ [5]

(b) Find the maximum volume of the cylinder which can be inscribed in a sphere of radius $3\sqrt{3}\ cm$. (Leave the answer in terms of π) [5]

Question 6

(a) Evaluate: $\int \frac{\cos^{-1} x}{x^2} dx$ [5]

(b) Find the area bounded by the curve $y = 2x - x^2$, and the line $y = x$. [5]

Question 7

(a) Find the Karl Pearson's co-efficient of correlation between x and y for the following data: [5]

x	16	18	21	20	22	26	27	15
y	22	25	24	26	25	30	33	14

(b) The following table shows the mean and standard deviation of the marks of Mathematics and Physics scored by the students in a school: [5]

	Mathematics	Physics
Mean	84	81
Standard Deviation	7	4

The correlation co-efficient between the given marks is 0·86. Estimate the likely marks in Physics if the marks in Mathematics are 92.

Question 8

(a) Bag A contains three red and four white balls; bag B contains two red and three white balls. If one ball is drawn from bag A and two balls from bag B, find the probability that: [5]

(i) One ball is red and two balls are white;

(ii) All the three balls are of the same colour.

(b) Three persons, Aman, Bipin and Mohan attempt a Mathematics problem independently. The odds in favour of Aman and Mohan solving the problem are 3:2 and 4:1 respectively and the odds against Bipin solving the problem are 2:1. Find: [5]

(i) The probability that all the three will solve the problem.

(ii) The probability that problem will be solved.

Question 9

(a) Find the locus of the complex number $z = x + iy$, satisfying relations $\arg(z-1) = \pi/4$ and $|z-2-3i| = 2$. Illustrate the locus on the Argand plane. [5]

(b) Solve the following differential equation: [5]

$ye^y\,dx = (y^3+2xe^y)\,dy$, given that $x = 0,\ y = 1$.

SECTION B

Question 10

(a) If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, then show that [5]

$|\vec{a}-\vec{b}| = 2\sin\frac{\theta}{2}$.

(b) Find the value of λ for which the four points A, B, C, D with position vectors $-\hat{j}-\hat{k}$; $4\hat{i}+5\hat{j}+\lambda\hat{k}$; $3\hat{i}+9\hat{j}+4\hat{k}$ and $-4\hat{i}+4\hat{j}+4\hat{k}$ are coplanar. [5]

Question 11

(a) Find the equation of a line passing through the point $(-1, 3, -2)$ and perpendicular to the lines: $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$. [5]

(b) Find the equations of planes parallel to the plane $2x - 4y + 4z = 7$ and which are at a distance of five units from the point $(3, -1, 2)$. [5]

Question 12

(a) If the sum and the product of the mean and variance of a Binomial Distribution are 1·8 and 0·8 respectively, find the probability distribution and the probability of at least one success. [5]

(b) For A, B and C, the chances of being selected as the manager of a firm are 4 : 1 : 2, respectively. The probabilities for them to introduce a radical change in the marketing strategy are 0·3, 0·8 and 0·5 respectively. If a change takes place; find the probability that it is due to the appointment of B. [5]

SECTION C

Question 13

(a) If Mr. Nirav deposits ₹250 at the beginning of each month in an account that pays an interest of 6% per annum compounded monthly, how many months will be required for the deposit to amount to at least ₹6,390 ? [5]

(b) A mill owner buys two types of machines A and B for his mill. Machine A occupies 1000 sqm of area and requires 12 men to operate it; while machine B occupies 1200 sqm of area and requires 8 men to operate it. The owner has 7600 sqm of area available and 72 men to operate the machines. If machine A produces 50 units and machine B produces 40 units daily, how many machines of each type should he buy to maximise the daily output? Use Linear Programming to find the solution. [5]

Question 14

(a) A bill of ₹60,000 was drawn on 1st April 2011 at 4 months and discounted for ₹58,560 at a bank. If the rate of interest was 12% per annum, on what date was the bill discounted? [5]

(b) A company produces a commodity with ₹24,000 fixed cost. The variable cost is estimated to be 25% of the total revenue recovered on selling the product at a rate of ₹8 per unit. Find the following: [5]

(i) Cost function

(ii) Revenue function

(iii) Breakeven point.

Question 15

(a) The price index for the following data for the year 2011 taking 2001 as the base year was 127. The simple average of price relatives method was used. Find the value of x: [5]

Items	A	B	C	D	E	F
Price (₹ per unit) in year 2001	80	70	50	20	18	25
Price (₹per unit) in year 2011	100	87·50	61	22	x	32·50

(b) The profits of a paper bag manufacturing company (in lakhs of rupees) during each month of a year are: [5]

Month	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Profit	1·2	0·8	1·4	1·6	2·0	2·4	3·6	4·8	3·4	1·8	0·8	1·2

Plot the given data on a graph sheet. Calculate the four monthly moving averages and plot these on the same graph sheet.